Algorithm for the calculation of vibration inherent frequencies bending from two-shafts transmission with technological deviations

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Abstract. Due to the inaccuracy of execution and assembly of the element cardan transmissions appear more disruptive forces that require the transmission. Under the action of these forces, the cardan transmission, which has a small section and a great length, during operation has the tendency to enter into vibration. If the operating speed coincides or is close to an own oscillation frequencies the resonance phenomenon occurs. The amplitude of the oscillations increase sharply, which can lead to fatigue fracture, malfunction, wear, vibration, noise or to itscomplete complete destruction. In this paper It is presented the algorithm for calculating inherent vibration frequencies and inherent mod of vibration at bending from two-shafts transmission with technological deviations.

1 Introduction

The technological deviations of execution and assembly of cardan transmissions, inserts further efforts, I. Bulac [1] in the kinematik pairs of cardan joints mechanisms. In general system of reference with OX_0 axis coincides with rotation axis, the efforts harmonic ranging Fl. Dudita [2], Fl. Dudita, D. Diaconescu, Cr. Bohn, M. Neagoe, and R. Saulescu [3], being permanent disturbance sources. The component acting on the perpendicularly direction on the rotation axis, leads to the change of inherent frequencies at bending. Taking as a basis a fixed real two-shafts transmission, with a technological deviation at one of the spindle of the cardan cross and starting from the dynamic model with distributed mass, Fl. Dudita, D. Diaconescu, Cr. Bohn, M. Neagoe, and R. Saulescu [3], A. Ripianu and I. Craciun [4], R. Voinea, D. Voiculescu and FL. Simion [5], N. Pandrea, S. Parlac and D. Popa [6], is developed a algorithm to determinate of the frequencies and vibration modes inherent at bending.

2 Mathematical model

The constructive solution for fixe two-shafts transmission (see Fig. 1.a.), is associated Fl. Dudita [2], Fl. Dudita, D. Diaconescu, Cr. Bohn, M. Neagoe, and R. Saulescu [3], the equivalent mechanical model (see Fig. 1.b.), the sections A and D elastic supports are located elastic constants k_A, k_D and the harmonic exciting forces R_A, R_D with the amplitudes $\tilde{R}_A, \tilde{R}_D$, activating in the sections A, D, which in the new system chosen acts on the direction axis Oy.

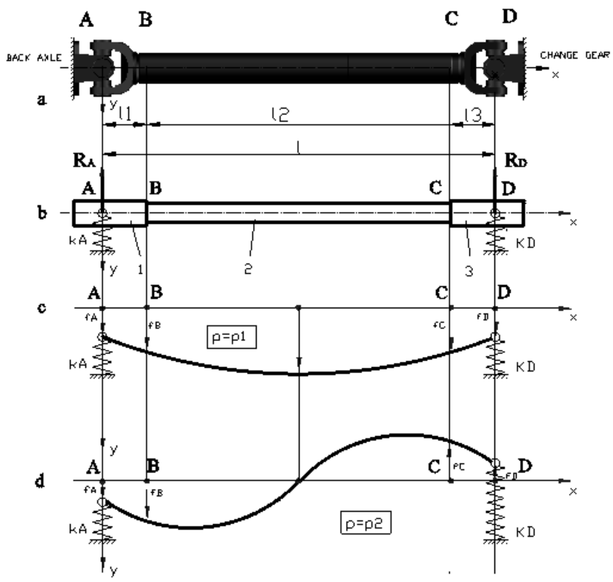

Fig. 1. Constructive model and equivalent mechanical model.

Next the notations are used A. Ripianu and I. Craciun [4], R. Voinea, D. Voiculescu and FL. Simion [5]

1) f_i, θ_i, M_i, F_i as the deflection, rotation, deflection moment and the cutting force from the
2) current section Δ_i ; Δ_A ; Δ_B ; Δ_C ; Δ_D state vectors, defined by the relations:

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$$\begin{aligned}\Delta_i &= (f_i, \theta_i, M_i, F_i)^T \\ \Delta_A &= (f_A, \theta_A, M_A, F_A)^T \\ \Delta_B &= (f_B, \theta_B, M_B, F_B)^T \\ \Delta_C &= (f_C, \theta_C, M_C, F_C)^T \\ \Delta_D &= (f_D, \theta_D, M_D, F_D)^T.\end{aligned} \quad (1)$$

3) $x_i, \rho_i, A_i, E_i, I_{yi}$ - respectively, the length, density, area, transverse modulus of elasticity and geometrical moment of inertia mainly for the section corresponding to the index $i = 1,2,3$.

4) The parameters α_i defined by the relations:

$$\alpha_i = \sqrt[4]{p^2 \frac{\rho_i A_i}{E_i Iyi}}. \quad (2)$$

where p is the vibration inherent pulse.

5) The parameters z_i defined by the relations:

$$z_i = \alpha_i \cdot x_i. \quad (3)$$

6) chz, shz - *sin* and cos the hyperbolic functions:

$$ch(z_i) = \frac{e^{z_i} + e^{-z_i}}{2} \;; sh(z_i) = \frac{e^{z_i} - e^{-z}{}_i}{2}. \quad (4)$$

7) $f_j(z_i)$, j=1,2,3,4 the Krâlov functions defined by relations:

$$\begin{aligned} f_1(z_i) &= \frac{ch(z_i) + cos(z_i)}{2}; \\ f_2(z_i) &= \frac{sh(z_i) + sin(z_i)}{2}; \\ f_3(z_i) &= \frac{ch(z_i) - cos(z_i)}{2}; \\ f_4(z_i) &= \frac{sh(z_i) - sin(z_i)}{2}. \end{aligned} \quad (5)$$

8) $F(z_i)$ –the Krâlov matrixes defined by relations:

$$F(z) = \begin{pmatrix} f_1(z) & f_2(z) & f_3(z) & f_4(z) \\ f_4(z) & f_1(z) & f_2(z) & f_3(z) \\ f_3(z) & f_4(z) & f_1(z) & f_2(z) \\ f_2(z) & f_3(z) & f_4(z) & f_1(z) \end{pmatrix}. \quad (6)$$

9) α , α^{-1} – for diagonal matrixes:

$$\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\alpha} & 0 & 0 \\ 0 & 0 & -\frac{1}{\alpha^2 EI_w} & 0 \\ 0 & 0 & 0 & -\frac{1}{\alpha^3 EI_w} \end{pmatrix} \quad (7)$$

$$\alpha^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & -\alpha^2 EI_w & 0 \\ 0 & 0 & 0 & -\alpha^3 EI_w \end{pmatrix}.$$

10) $R_i, i - 1,2,3$, the field matrixes of sections:

$$R_i = \alpha_i^{-1} \cdot F(\alpha_i x_i) \cdot \alpha_i. \quad (8)$$

11) R, the field matrix for all two-shafts transmission limited points A, D:

$$R = R_3 \cdot R_2 \cdot R_1. \quad (9)$$

With these notations, in the absence of the disturbing forces, is obtained A. Ripianu and I. Craciun [4], R. Voinea, D. Voiculescu and FL. Simion [5] the equality:

$$\Delta_D = R \cdot \Delta_A. \quad (10)$$

and in the presence of disturbing forces is obtained:

$$\Delta_D + \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\tilde{R}_D \end{pmatrix} = R \cdot \Delta_A + R \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_A \end{pmatrix}. \quad (11)$$

By complying the bonds from sections A and D with the articulations results that:

$$\begin{aligned} M_A &= M_D = 0 \\ F_A &= k_A f_A \\ F_D &= -k_D f_D. \end{aligned} \quad (12)$$

and then the equality (11) in the condition of harmonic excitations becomes:

$$\begin{pmatrix} \tilde{f}_D \\ \tilde{\theta}_D \\ 0 \\ -k_D \tilde{f}_D \end{pmatrix} - R \cdot \begin{pmatrix} \tilde{F}_A / k_A \\ \tilde{\theta}_A \\ 0 \\ \tilde{F}_A \end{pmatrix} = R \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_A \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_D \end{pmatrix}. \quad (13)$$

where by $\tilde{f}_D, \tilde{\theta}_D, \tilde{\theta}_A, \tilde{f}_A$ were noted the amplitudes of sizes $f_D, \theta_D, \theta_A, f_A$.

Next, using the notations:

$$\mathbf{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\mathbf{k_D} & 0 & 0 & 0 \end{pmatrix} - \mathbf{R} \cdot \begin{pmatrix} 0 & 0 & 0 & 1/\mathbf{k_A} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (14)$$

and

$$\mathbf{F}^* = \mathbf{R} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{\mathbf{R}}_\mathbf{A} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{\mathbf{R}}_\mathbf{D} \end{pmatrix}. \quad (15)$$

from (13) is obtained the equality

$$\mathbf{R}^* \cdot \begin{pmatrix} \tilde{\mathbf{f}}_\mathbf{D} \\ \tilde{\theta}_\mathbf{D} \\ \tilde{\theta}_\mathbf{A} \\ \tilde{\mathbf{F}}_\mathbf{A} \end{pmatrix} = \mathbf{F}^*; \begin{pmatrix} \tilde{\mathbf{f}}_\mathbf{D} \\ \tilde{\theta}_\mathbf{D} \\ \tilde{\theta}_\mathbf{A} \\ \tilde{\mathbf{F}}_\mathbf{A} \end{pmatrix} = \mathbf{R}^{*-1}\mathbf{F}^*. \quad (16)$$

and hence the homogeneous system of equations is obtained in $\tilde{\theta}_A; \tilde{F}_A; \tilde{\theta}_D; \tilde{f}_D$.

The state vectors sizes amplitudes from sections B, C are given by the relations:

$$\begin{pmatrix} \tilde{\mathbf{f}}_\mathbf{B} \\ \tilde{\theta}_\mathbf{B} \\ \tilde{\mathbf{M}}_\mathbf{B} \\ \tilde{\mathbf{F}}_\mathbf{B} \end{pmatrix} = \mathbf{R}_1 \cdot \begin{pmatrix} \tilde{\mathbf{F}}_\mathbf{A}/\mathbf{k} \\ \tilde{\theta}_\mathbf{A} \\ 0 \\ \tilde{\mathbf{F}}_\mathbf{A} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{\mathbf{R}}_\mathbf{A} \end{pmatrix}$$
$$\begin{pmatrix} \tilde{\mathbf{f}}_\mathbf{C} \\ \tilde{\theta}_\mathbf{C} \\ \tilde{\mathbf{M}}_\mathbf{C} \\ \tilde{\mathbf{F}}_\mathbf{C} \end{pmatrix} = \mathbf{R}_2 \cdot \begin{pmatrix} \tilde{\mathbf{f}}_\mathbf{B} \\ \tilde{\theta}_\mathbf{B} \\ \tilde{\mathbf{M}}_\mathbf{B} \\ \tilde{\mathbf{F}}_\mathbf{B} \end{pmatrix}. \quad (17)$$

3 The calculation algorithm

For the numerical calculation one uses the following algorithm:

1) The following parameters are considered as known: $k_A, k_D, A_i, \rho_i, E_i, I_i, l_i \quad i = 1,2,3$
2) A value is chosen for the pulsation p and a variation step Δp ;
3) Calculate parameters $\alpha_i, i = 1,2,3$ with relation (2) and matrixes α , α^{-1} with relations (7) ;
4) The parameters z_i and hyperbolic functions $chz_i, shz_i, i = 1,2,3$ using relations (3) and (4) ;
5) Calculate Krâlov functions $f_j(z_i), j = 1,2,3,4; i = 1,2,3$, and Krâlov matrixes $F(z_i)$, with relations (5) and (6) ;
6) Matrixes calculus, $R_i, i-1,2,3$ and R with relations (8) and (9);
7) The calculation algorithm presented in the paper I. Bulac [1] is also calculated, the excitation forces $\tilde{R}_A, \tilde{R}_D$ given by the technical deviations, in the general reference system $OX_0Y_0Z_0$, from which are retained their maximum amplitudes;
8) Matrixes calculus, R^* and F^* with relations (14), (15) ;
9) The column matrix $\left(\tilde{f}_D, \tilde{\theta}_D, \tilde{\theta}_A, \tilde{F}_A\right)^T$ is calculated with the relation (16) ;
10) The value of deflection from A is also calculated with the relation $\tilde{f}_A = \tilde{F}_A / k_A$;
11) p is replaced with $p + \Delta p$ and the calculations are made again until the deflection from A reaches resonance (the value converges to $\propto$) , so being obtained the own pulsations $p_1, p_2, \ldots$;
12) The column matrix of the state vector from point D is calculated for the values of the own pulsations $p_1, p_2, \ldots$ in which the deflection from D is calculated with the relation $\tilde{f}_D = \tilde{F}_D / k_D$ and $M_D = 0$;
13) The column matrixes of state vectors from points B and C are calculated for the values of the own with the relations (17) and the deflections $\tilde{f}_B, \tilde{f}_C$ are extracted;
14) Corresponding to the deflections values for the own pulsations, it is established and marked the own vibrations.

4 Numerical application

Consider a mobile two-shafts transmission equipping a SUV vehicle whose construction model is shown in Fig. 1., for the following construction features that are known and mechanical:

$k_A = 85 \cdot 10^6 (N/m); k_D = 20 \cdot 10^6 (N/m); l_1 = l_3 = 0{,}1(m);$
$l_2 = 0{,}8(m); A_1 = A_3 = 19{,}6 \cdot 10^{-4}(m^2); A_2 = 3{,}6 \cdot 10^{-4}(m^2);$
$\rho_1 = \rho_2 = \rho_3 = 7800(kg/m^3)$

with a local deviation from perpendicularity of 0.0001 (m) at one of the spindle of the cardan cross of the cardam joint mechanism from the transmission of the component.

Based on the presented algorithm and a computer program developed in Excel or obtained first and second pulsation own values p_1=774,412(s^{-1}), and p_2=2909,1148(s^{-1}). According to the first way of vibration graphs were drawn at the bending vibration inherent mode Fig. 2.and the variation modes of the amplitudes in the zone of pulses of resonance for the considered points A,B,C and D, Fig. 3, Fig. 4, Fig. 5 and Fig. 6.

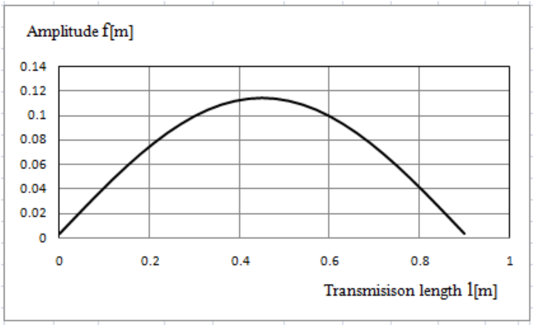

Fig. 2. Vibration mode corresponding to the first inherent pulse

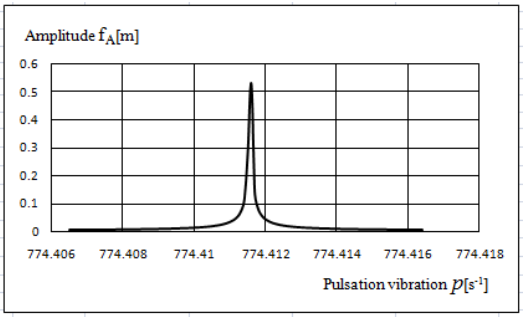

Fig. 3. Variation of the amplitude for the first inherent pulse from the point A.

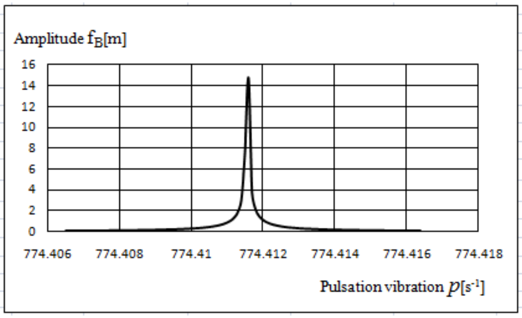

Fig. 4. Variation of the amplitude for the first inherent pulse from the point B.

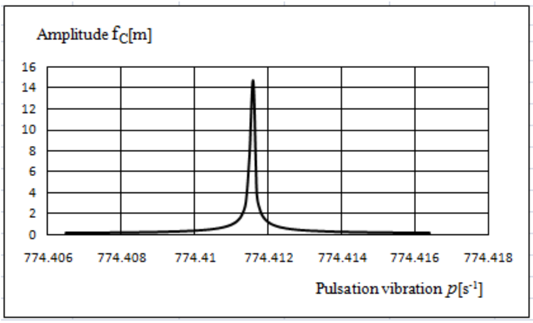

Fig. 5. Variation of the amplitude for the first inherent pulse from the point C.

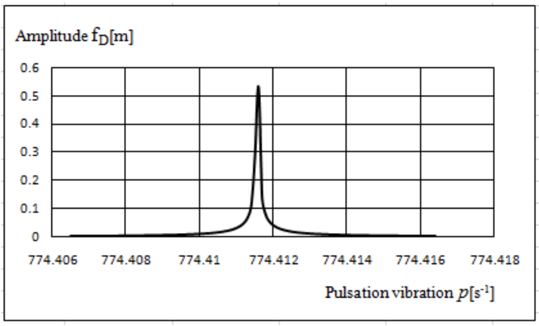

Fig. 6. Variation of the amplitude for the first inherent pulse from the point D.

5 Conclusions

The algorithm presented together with the computer program makes is possible the study of the own frequencies and vibration modes at bending of the two-shafts transmission with technical deviations at the component elements. Unlike the two-ideal transmission shafts transmission, where the mathematical model and calculation algorithm determines the own pulsations, starting from a value for the pulsation p after which p is replaced with $p+\Delta p$ and the calculations are made again, up when the characteristic equation (17) is satisfied, in the case of the the two-shafts transmission with technical deviations starts from a value for the pulsation p after which is replaced with $p+\Delta p$ and the calculations are made again until the deflection from A reaches resonance (the value converges to $\propto$).

References

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